Triple Coordinate Transforms for Prediction of Falling Cylinder Through the Water Column

Peter C. Chu, Chenwu Fan, Ashley D. Evans, and Anthony Gilles
Department of Oceanography, Naval Postgraduate School, 833 Dyer Road, Monterey, CA 93943

Triple coordinate systems are introduced to predict translation and orientation of falling rigid cylinder through the water column: earth-fixed coordinate (E-coordinate), cylinder's main-axis following coordinate (M-coordinate), and hydrodynamic force following coordinate (F-coordinate). Use of the triple coordinate systems and the transforms among them leads to the simplification of the dynamical system. The body and buoyancy forces and their moments are easily calculated using the E-coordinate system. The hydrodynamic forces (such as the drag and lift forces) and their moments are easily computed using the F-coordinate. The cylinder's moments of gyration are simply represented using the M-coordinate. Data collected from a cylinder-drop experiment at the Naval Postgraduate School swimming pool in June 2001 show great potential of using the triple coordinate transforms. [DOI: 10.1115/1.1651093]

1 Introduction

Consider an axially symmetric cylinder with the centers of mass (**X**) and volume (**B**) on the main axis (Fig. 1). Let (L,d,χ) represent the cylinder's length, diameter, and the distance between the two points (**X**,**B**). The positive χ-values refer to nose-down case, i.e., the center of mass (COM) is lower than the center of volume (COV). Three coordinate systems are used to model the hydrodynamics of falling cylinder through the water column: earth-fixed coordinate (E-coordinate), cylinder's main-axis following coordinate (M-coordinate), and hydrodynamic force following coordinate (F-coordinate). All the systems are three-dimensional, orthogonal, and right-handed.

2 Triple Coordinate Systems

2.1 E-Coordinate. The E-coordinate is represented by $F_E(O,\mathbf{i},\mathbf{j},\mathbf{k})$ with the origin "O," and three axes: *x*, *y*-axes (horizontal) with the unit vectors (**i**,**j**) and z-axis (vertical) with the unit vector **k** (upward positive). The position of the cylinder is represented by the position of the COM,

$$\mathbf{X}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}, \tag{1}$$

which is translation of the cylinder. The translation velocity is given by

$$\frac{d\mathbf{X}}{dt}=\mathbf{V}, \quad \mathbf{V}=(u,v,w). \tag{2}$$

2.2 M-Coordinate. Let orientation of the cylinder's main-axis (pointing downward) is given by $\mathbf{i}_M$. The angle between $\mathbf{i}_M$ and **k** is denoted by $\psi_2+\pi/2$. Projection of the vector $\mathbf{i}_M$ onto the (x,y) plane creates angle (ψ_3) between the projection and the *x*-axis (Fig. 2). The M-coordinate is represented by $F_M(\mathbf{X},\mathbf{i}_M,\mathbf{j}_M,\mathbf{k}_M)$ with the origin "**X**," unit vectors $(\mathbf{i}_M,\mathbf{j}_M,\mathbf{k}_M)$, and coordinates (x_M,y_M,z_M). In the plane consisting of vectors $\mathbf{i}_M$ and **k** (passing through the point *M*, called the IMK plane), two new unit vectors $(\mathbf{j}_M,\mathbf{k}_M)$ are defined with $\mathbf{j}_M$ perpendicular to the IMK plane, and $\mathbf{k}_M$ perpendicular to $\mathbf{i}_M$ in the IMK plane. The unit vectors of the M-coordinate system are given by (Fig. 2)

$$\mathbf{j}_M=\mathbf{k}\times\mathbf{i}_M, \quad \mathbf{k}_M=\mathbf{i}_M\times\mathbf{j}_M. \tag{3}$$

The M-coordinate system is solely determined by orientation of the cylinder's main-axis $\mathbf{i}_M$. Let the vector **P** be represented by ${}^E\mathbf{P}$ in the E-coordinate and by ${}^M\mathbf{P}$ in the M-coordinate, and let ${}^E_M\mathbf{R}$ be the rotation matrix from the M-coordinate to the E-coordinate,

$${}^E_M\mathbf{R}(\psi_2,\psi_3)\equiv\begin{bmatrix} r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33}\end{bmatrix}=\begin{bmatrix}\cos\psi_3 & -\sin\psi_3 & 0\\ \sin\psi_3 & \cos\psi_3 & 0\\ 0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}\cos\psi_2 & 0 & \sin\psi_2\\ 0 & 1 & 0\\ -\sin\psi_2 & 0 & \cos\psi_2\end{bmatrix}, \tag{4}$$

which represents $(\mathbf{i}_M,\mathbf{j}_M,\mathbf{k}_M)$,

$$\mathbf{i}_M=\begin{bmatrix} r_{11}\\ r_{21}\\ r_{31}\end{bmatrix}, \quad \mathbf{j}_M=\begin{bmatrix} r_{12}\\ r_{22}\\ r_{32}\end{bmatrix}, \quad \mathbf{k}_M=\begin{bmatrix} r_{13}\\ r_{23}\\ r_{33}\end{bmatrix}. \tag{5}$$

Transformation of ${}^M\mathbf{P}$ into ${}^E\mathbf{P}$ contains rotation and translation,

$${}^E\mathbf{P}={}^E_M\mathbf{R}(\psi_2,\psi_3)\,{}^M\mathbf{P}+\mathbf{X}. \tag{6}$$

Let the cylinder rotate around $(\mathbf{i}_M,\mathbf{j}_M,\mathbf{k}_M)$ with angles $(\varphi_1,\varphi_2,\varphi_3)$ (Fig. 2). The angular velocity of cylinder is calculated by

$$\omega_1=\frac{d\varphi_1}{dt}, \quad \omega_2=\frac{d\varphi_2}{dt}, \quad \omega_3=\frac{d\varphi_3}{dt}, \tag{7}$$

and

$$\psi_1=\varphi_1, \quad \frac{d\psi_2}{dt}=\frac{d\varphi_2}{dt}=\omega_2, \quad \frac{d\psi_3}{dt}\neq\frac{d\varphi_3}{dt}. \tag{8}$$

If $(\omega_1,\omega_2,\omega_3)$ are given, time integration of (7) with the time interval Δt leads to

$$\Delta\varphi_1=\omega_1\Delta t, \quad \Delta\varphi_2=\omega_2\Delta t, \quad \Delta\varphi_3=\omega_3\Delta t. \tag{9}$$

The increments $(\Delta\psi_2,\Delta\psi_3)$ are determined by the relationship between the two rotation matrices ${}^E_MR(\psi_2+\Delta\psi_2,\psi_3+\Delta\psi_3)$ and ${}^E_MR(\psi_2,\psi_3)$

$${}^E_MR(\psi_2+\Delta\psi_2,\psi_3+\Delta\psi_3)={}^E_MR(\psi_2,\psi_3)\begin{bmatrix}\cos(\Delta\varphi_3) & -\sin(\Delta\varphi_3) & 0\\ \sin(\Delta\varphi_3) & \cos(\Delta\varphi_3) & 0\\ 0 & 0 & 1\end{bmatrix}\times\begin{bmatrix}\cos(\Delta\varphi_2) & 0 & \sin(\Delta\varphi_2)\\ 0 & 1 & 0\\ -\sin(\Delta\varphi_2) & 0 & \cos(\Delta\varphi_2)\end{bmatrix}. \tag{10}$$

2.3 F-Coordinate. The F-coordinate is represented by $F_F(\mathbf{X},\mathbf{i}_F,\mathbf{j}_F,\mathbf{k}_F)$ with the origin **X**, unit vectors $(\mathbf{i}_F,\mathbf{j}_F,\mathbf{k}_F)$, and coordinates (x_F,y_F,z_F). Let $\mathbf{V}_w$ be the fluid velocity. The water-to-cylinder velocity is represented by $\mathbf{V}_r=\mathbf{V}_w-\mathbf{V}$, that is decomposed into two parts,

$$\mathbf{V}_r=\mathbf{V}_1+\mathbf{V}_2, \quad \mathbf{V}_1=(\mathbf{V}_r\cdot\mathbf{i}_F)\mathbf{i}_F, \quad \mathbf{V}_2=\mathbf{V}_r-(\mathbf{V}_r\cdot\mathbf{i}_F)\mathbf{i}_F, \tag{11}$$

Contributed by the Applied Mechanics Division of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS for publication in the ASME JOURNAL OF APPLIED MECHANICS. Manuscript received by the ASME Applied Mechanics Division, May 14, 2003; final revision, Sept. 19, 2003. Associate Editor: D. A. Siginer.

PROOF COPY 004402AMJ

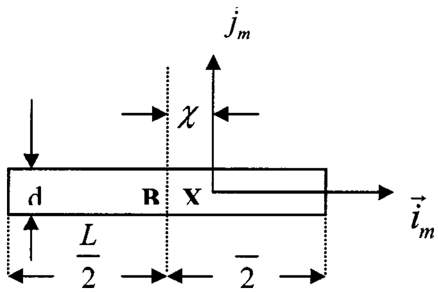

Fig. 1 M-coordinate with the COM as the origin X and $(\mathbf{i}_m, \mathbf{j}_m)$ as the two axes. Here, χ is the distance between the COV (B) and COM, (L, d) are the cylinder's length and diameter.

where $\mathbf{V}_1$ is the component paralleling to the cylinder's main-axis (i.e., along $\mathbf{i}_\mathrm{M}$), and $\mathbf{V}_2$ is the component perpendicular to the cylinder's main-axial direction. The unit vectors for the F-coordinate are defined by (column vectors)

$$\mathbf{i}_F = \mathbf{i}_M = \begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \end{bmatrix}, \quad \mathbf{j}_\mathrm{F} = \mathbf{V}_2 / |\mathbf{V}_2|, \quad \mathbf{k}_\mathrm{F} = \mathbf{i}_\mathrm{F} \times \mathbf{j}_\mathrm{F}. \tag{12}$$

The F-coordinate system is solely determined by orientation of the cylinder's main-axis ($\mathbf{i}_\mathrm{M}$) and the water-to-cylinder velocity. Note that the M and F-coordinate systems have one common unit vector $\mathbf{i}_\mathrm{M}$ (orientation of the cylinder).

Let ${}^E_F\mathbf{R}$ be the rotation matrix from the F-coordinate to the E-coordinate,

$${}^E_F\mathbf{R}(\psi_2, \psi_3, \phi_{MF}) \equiv \begin{bmatrix} r_{11} & r'_{12} & r'_{13} \\ r_{21} & r'_{22} & r'_{23} \\ r_{31} & r'_{32} & r'_{33} \end{bmatrix}, \quad \phi_{MF} \equiv (\mathbf{j}_M, \mathbf{j}_F), \tag{13}$$

which leads to

$$\mathbf{i}_F = \begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \end{bmatrix}, \quad \mathbf{j}_F = \begin{bmatrix} r'_{12} \\ r'_{22} \\ r'_{32} \end{bmatrix}, \quad \mathbf{k}_F = \begin{bmatrix} r'_{13} \\ r'_{23} \\ r'_{33} \end{bmatrix}. \tag{14}$$

Here, ϕ_{MF} is the angle between the two unit vectors $(\mathbf{j}_M, \mathbf{j}_F)$. Let the vector $\mathbf{P}$ be represented by ${}^F\mathbf{P}$ in the F-coordinate. Transformation of ${}^F\mathbf{P}$ into ${}^E\mathbf{P}$ contains rotation and translation,

$${}^E\mathbf{P} = {}^E_F\mathbf{R}(\psi_2, \psi_3, \phi_{MF})\,{}^F\mathbf{P} + \mathbf{X}. \tag{15}$$

Use of the F-coordinate system simplifies the calculations for the lift and drag forces and torques acting on the cylinder. Since the M and F-coordinates share a common axis $\mathbf{i}_\mathrm{M} = \mathbf{i}_\mathrm{F}$, the rotation matrix from the F to M-coordinate systems is given by

$$\begin{aligned} {}^M_F\mathbf{R} &= {}^M_E\mathbf{R}\,{}^F_E\mathbf{R} = {}^E_M\mathbf{R}^{-1}(\psi_2, \psi_3)\,{}^E_F\mathbf{R}(\psi_2, \psi_3, \phi_{MF}) \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & e_{22} & e_{23} \\ 0 & e_{32} & e_{33} \end{bmatrix}, \end{aligned} \tag{16}$$

is two-dimensional with the rotation matrix given by

$${}^M_F\mathbf{E} = [\mathbf{e}_2 \;\; \mathbf{e}_3], \quad \mathbf{e}_2 = \begin{bmatrix} e_{22} \\ e_{32} \end{bmatrix}, \quad \mathbf{e}_3 = \begin{bmatrix} e_{23} \\ e_{33} \end{bmatrix}. \tag{17}$$

Let the cylinder rotate around $(\mathbf{i}_\mathrm{F}, \mathbf{j}_\mathrm{F}, \mathbf{k}_\mathrm{F})$ with the angular velocity components represented by $(\omega'_1, \omega'_2, \omega'_3)$ (Fig. 2). They are connected to the angular velocity components in the M-coordinate system by

$$\omega'_1 = \omega_1, \quad \begin{bmatrix} \omega'_2 \\ \omega'_3 \end{bmatrix} = {}^F_M\mathbf{E} \begin{bmatrix} \omega_2 \\ \omega_3 \end{bmatrix}. \tag{18}$$

3 Prediction of Hydrodynamic Characteristics of Falling Cylinder

3.1 Translation Velocity. The translation velocity of the cylinder ($\mathbf{V}$) is governed by the momentum equation in the E-coordinate system,

$$\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ (1 - \rho_w / \overline{\rho})g \end{bmatrix} + \frac{1}{\overline{\rho}\Pi}\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}, \tag{19}$$

where g is the gravitational acceleration; $\overline{\rho}$ is the average cylinder density; ρ_w is the water density; Π is the cylinder volume; and $\overline{\rho}\Pi = m$, is the cylinder mass; (F_x, F_y, F_z) are the hydrodynamic force (including drag and lift forces) components. The drag and lift forces are calculated using the drag and lift laws with the given water-to-cylinder velocity ($\mathbf{V}_r$) that is calculated using the F-coordinate.

3.2 Cylinder's Orientation. It is convenient to write the moment of momentum equation

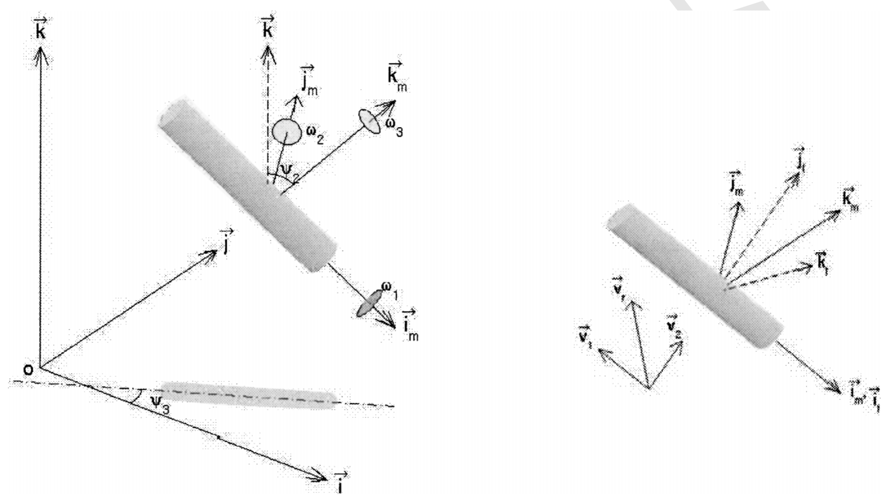

Fig. 2 Three coordinate systems

$$\mathbf{J}\cdot\frac{d\boldsymbol{\omega}}{dt}=\mathbf{M}_b+\mathbf{M}_h, \tag{20}$$

in the M-coordinate system with the cylinder's angular velocity components $(\omega_1,\omega_2,\omega_3)$ defined by (7). Here, $\mathbf{M}_b$ and $\mathbf{M}_h$ are the body and surface force torques. The moment of gyration tensor for the axially symmetric cylinder is a diagonal matrix

$$\mathbf{J}=\begin{bmatrix} J_1 & 0 & 0\\ 0 & J_2 & 0\\ 0 & 0 & J_3\end{bmatrix}, \tag{21}$$

where J_1, J_2, and J_3 are the moments of inertia. The gravity force, passing the COM, doesn't induce the moment. The buoyancy force induces the moment in the $\mathbf{j}_{\mathrm{M}}$ direction if the COM doesn't coincide with the COV (i.e., $\chi\neq 0$),

$$\mathbf{M}_b=\Pi\chi\rho_w g\cos\psi_2\mathbf{j}_{\mathrm{M}}. \tag{22}$$

The moment of the hydrodynamic force in $\mathbf{i}_{\mathrm{F}}$ direction is not caused by the drag and lift forces, but by the viscous fluid. The moment of the viscous force is calculated by (White [1])

$$\mathbf{M}_{v1}=-C_{m1}\omega_1\mathbf{i}_{\mathrm{F}},\quad C_{m1}\equiv\pi\mu L d^2. \tag{23}$$

When the cylinder rotates around $\mathbf{j}_F$ with the angular velocity ω_2', the drag force exerts the torque on the cylinder in the $\mathbf{j}_{\mathrm{F}}$ direction ($\mathbf{M}_{d2}$) and in the $\mathbf{k}_{\mathrm{F}}$ direction ($\mathbf{M}_{d3}$). The lift force exerts the torque on the cylinder in the $\mathbf{j}_{\mathrm{F}}$ direction ($\mathbf{M}_{l2}$). The moment of hydrodynamic force $\mathbf{M}_h$

$$\mathbf{M}_h=\mathbf{M}_{v1}+\mathbf{M}_{d2}+\mathbf{M}_{d3}+\mathbf{M}_{l2} \tag{24}$$

is represented in M-coordinate. Note that the M and F-coordinate systems have the same x-axis, $\mathbf{i}_{\mathrm{M}}=\mathbf{i}_{\mathrm{F}}$. The equations for $(\omega_1,\omega_2,\omega_3)$ are given by

$$\frac{d\omega_1}{dt}=-a_1\omega_1, \tag{25}$$

$$\frac{d}{dt}\begin{bmatrix}\omega_2\\ \omega_3\end{bmatrix}=-\mathbf{B}\cdot\begin{bmatrix}\omega_2\\ \omega_3\end{bmatrix}+\boldsymbol{\alpha}_2, \tag{26}$$

where

$$a_1\equiv\frac{C_{m1}}{J_1}=8\pi\mu L/m,$$

$$\mathbf{B}\equiv\begin{bmatrix}\frac{1}{J_2} & 0\\ 0 & \frac{1}{J_3}\end{bmatrix}\cdot(C_{m2}\mathbf{e}_2\mathbf{e}_2^T+C_{m3}\mathbf{e}_3\mathbf{e}_3^T-C_{ml}\mathbf{e}_2\mathbf{e}_3^T),$$
$$\tag{27}$$
$$\boldsymbol{\alpha}_2\equiv\begin{bmatrix}\frac{1}{J_2} & 0\\ 0 & \frac{1}{J_3}\end{bmatrix}\cdot(M_1\mathbf{e}_2-M_3\mathbf{e}_3)+\frac{\Pi\chi g\rho_w}{J_2}\cos\psi_2\begin{bmatrix}1\\0\end{bmatrix}.$$

Here, $M_l\equiv 1/2d\rho_w/(1+f_r)LV_2^2\chi$, $M_3\equiv 1/2C_{d2}d\rho_w/(1+f_r)V_2^2L\chi$, and f_{r} is the added mass factor for the moment of drag and lift forces. Equation (25) has the analytical solution

$$\omega_1(t)=\omega_1(t_0)\exp[-a_1(t-t_0)], \tag{28}$$

which represents damping rotation of the cylinder around the main axis ($\mathbf{i}_{\mathrm{M}}$). The Euler-typed forward difference is used to solve the five Eqs. (19), (26), and (28).

4 Model Evaluation

The Cylinder Drop Experiment (CYDEX) was conducted at the Naval Postgraduate School (NPS) in July 2001 (Chu et al. [2]) to evaluate the three-dimensional theoretical model. It consisted of dropping cylinders whose physical conditions are illustrated in Table 1 into the water and recording the position as a function of time using two digital cameras at (30 Hz) as the cylinders fell 2.4 meters to the pool bottom. After analyzing the CODEX experimental data, seven general trajectory patterns (Table 2) are identified: straight, slant, spiral, flip, flat, see-saw, and combination (Fig. 3). Dependence of the trajectory patterns on the cylinders' physical parameters and release conditions are illustrated in Table 3. The theoretical model predicts the motion of cylinder inside the water column reasonably well. Two examples are listed for illustration.

Table 1 Physical parameters of the model cylinders

Cylinder	Mass (g)	L (cm)	Volume (cm^3)	ρ_m ($\mathrm{g\ m^{-3}}$)	J_1 ($\mathrm{g\ m^2}$)	χ (cm)	$J_2(J_3)$ ($\mathrm{g\ m^2}$)
	322.5	15.20	191.01	1.69	330.5	0.00	6087.9
1						0.74	5783.0
						1.48	6233.8
2	254.2	12.10	152.05	1.67	271.3	0.06	3424.6
						0.53	3206.5
						1.00	3312.6
3	215.3	9.12	114.61	1.88	235.0	0.00	1695.2
						0.29	1577.5
						0.58	1556.8

Positive χ (Nose-Down). Cylinder #1 ($L=15.20$ cm, $\overline{\rho}=1.69$ g cm^{-3}) with χ=0.74 m is injected to the water with the drop angle 45 deg. The physical parameters of this cylinder are given by

$$m=322.5\ \mathrm{g},\quad J_1=330.5\ \mathrm{g\ cm^2},\quad J_2=J_3=5783.0\ \mathrm{g\ cm^2}. \tag{29a}$$

Undersea cameras measure the initial conditions

$$x_0=0,\quad y_0=0,\quad z_0=0,\quad u_0=0,\quad v_0=-1.55\ \mathrm{m\ s^{-1}},$$
$$w_0=-2.52\ \mathrm{m\ s^{-1}}, \tag{29b}$$
$$\psi_{10}=0,\quad \psi_{20}=60\ \mathrm{deg},\quad \psi_{30}=-95\ \mathrm{deg},\quad \omega_{10}=0,$$
$$\omega_{20}=0.49\ \mathrm{s^{-1}},\quad \omega_{30}=0.29\ \mathrm{s^{-1}}.$$

Substitution of the model parameters (29a) and the initial conditions (29b) into the theoretical model ((19), (26), (28)) leads to the prediction of the cylinder's translation and orientation that are compared with the data collected during CYDEX at time steps (Fig. 4). Both model simulated and observed tracks show a slant-straight pattern.

Table 2 Trajectory patterns

Trajectory Pattern	Description
Straight	Cylinder exhibited little angular change about z-axis. The attitude remained nearly parallel with z-axis (±15 deg).
Slant	Cylinder exhibited little angular change about z-axis. The attitude was 45 deg off z-axis (±15 deg).
Spiral	Cylinder experienced rotation about z-axis throughout the water column
Flip	Initial water entry point rotated at least 180 deg
Flat	Cylinder's angle with vertical near 90 deg for most of the trajectory
Seesaw	Similar to the flat pattern except that cylinder's angle with vertical would oscillate between greater (less) than 90 deg and less (greater) than 90 deg like a seesaw
Combin-ation	Complex trajectory where cylinder exhibited several of theabove patterns

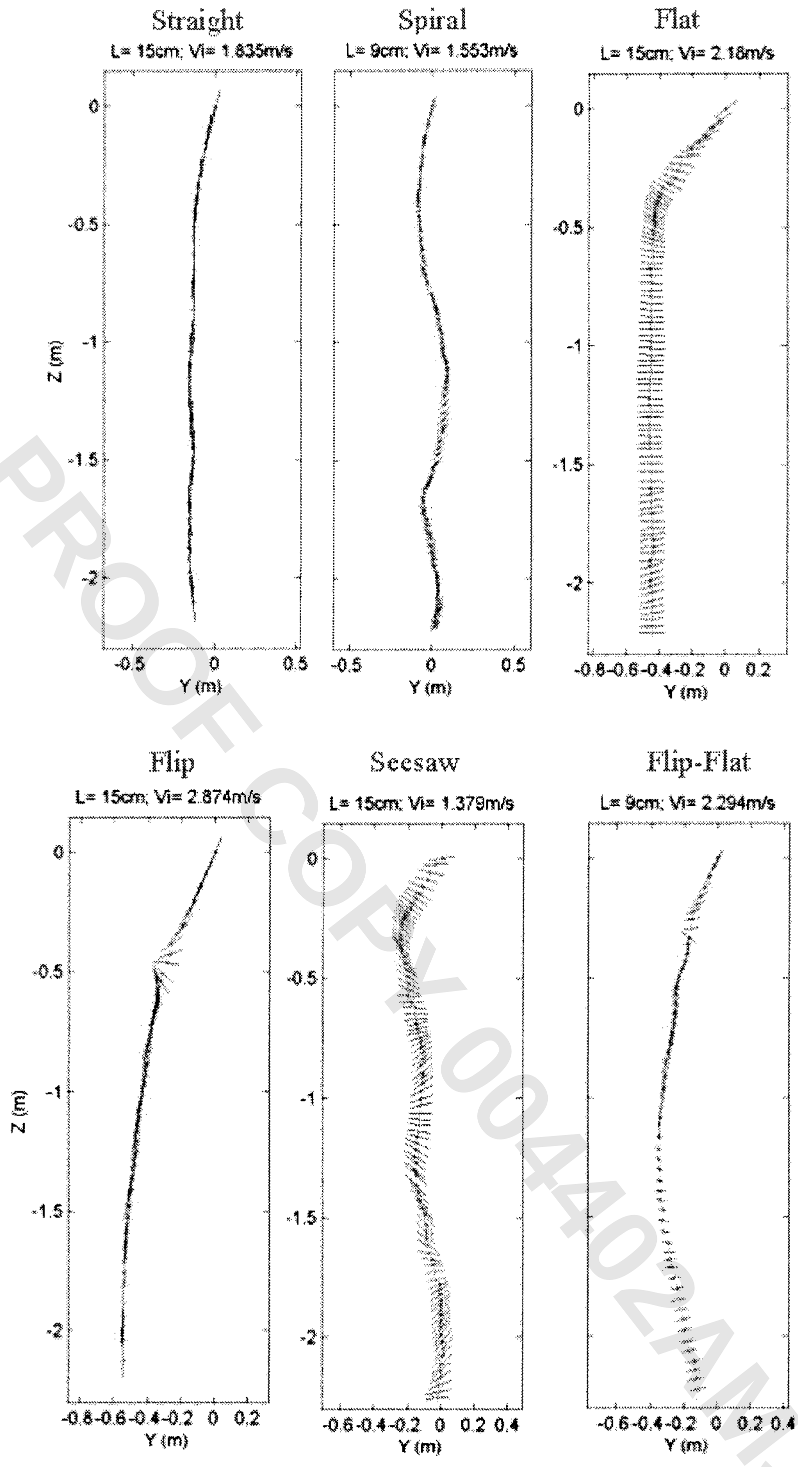

Fig. 3 Cylinders' track patterns observed during CYDEX

Negative χ (Nose-Up): Cylinder #2 ($L = 12.10$ cm, $\overline{\rho} = 1.67\ \mathrm{g\,cm^{-3}}$) with $\chi = -1.00$ cm is injected to the water with the drop angle 30 deg. The physical parameters of this cylinder are given by

$$m = 254.2\ \mathrm{g}, \quad J_1 = 271.3\ \mathrm{g\ cm^2}, \quad J_2 = J_3 = 3312.6\ \mathrm{g\ cm^2}. \tag{30a}$$

Undersea cameras measure the initial conditions

$$x_0 = 0, \quad y_0 = 0, \quad z_0 = 0, \quad u_0 = 0, \quad v_0 = -0.75\ \mathrm{m\ s^{-1}}, \quad w_0 = -0.67\ \mathrm{m\ s^{-1}}, \tag{30b}$$

$$\psi_{10} = 0, \quad \psi_{20} = 24\ \mathrm{deg}, \quad \psi_{30} = -96\ \mathrm{deg}, \quad \omega_{10} = 0, \quad \omega_{20} = -5.08\ \mathrm{s^{-1}}, \quad \omega_{30} = 0.15\ \mathrm{s^{-1}}.$$

The predicted cylinder's translation and orientation are compared with the data collected during CYDEX at time steps (Fig. 5). The simulated and observed tracks show flip-spiral pattern. The flip

Table 3 Trajectory patterns for nose-down dropping (χ>0)

Cylinder Length (cm) χ (cm)	15.20 1.48	12.10 1.00	9.12 0.58
Drop angle 15 deg	Straight (1) Slant-straight* (3)	Straight (1), Spiral (1) Slant-straight* (2)	Spiral* (2) Straight-slant (1) Slant-straight (1)
Drop angle 30 deg	Straight (1) Slant-straight* (4)	Slant (1), Spiral (1) Straight (1) Slant-straight* (2)	Spiral* (5)
Drop angle 45 deg	Slant* (2), Straight (1) Slant-straight (1) Straight-spiral (1)	Straight (1) Spiral* (2) Straight-spiral (1) Slant-straight (1)	Spiral* (4) Slant-spiral (1)
Drop angle 60 deg	Straight** (5)	Straight* (3) Straight-spiral (1) Straight-slant (1)	Spiral* (4) Straight-spiral (1)
Drop angle 75 deg	Straight** (5)	Straight (2) Straight-spiral (3)	Spiral (2), Slant (1) Straight-spiral (2)

occurs at 0.11 s (0.13 s) after cylinder entering the water in the experiment (model). After the flip, the cylinder spirals down to the bottom.

5 Conclusions

(1) Triple coordinate systems are suggested to predict the translation and orientation of falling rigid cylinder through water column: earth-fixed coordinate (E-coordinate), cylinder's main-axis following coordinate (M-coordinate), and hydrodynamic force following coordinate (F-coordinate). It simplifies the computation with the body and buoyancy forces and their moments in the E-coordinate system, the hydrodynamic forces (such as the drag and lift forces) and their moments in the F-coordinate, and the cylinder's moments of gyration in the M-coordinate.

(2) Usually, the momentum (moment of momentum) equation for predicting the cylinder's translation velocity (orientation) is represented in the E-coordinate (M-coordinate) system. Transformations among the three coordinate systems are used to convert the forcing terms into E-coordinate (M-coordinate) for the momentum (moment of momentum) equation. A numerical model is developed on the base of the triple coordinate transform to predict the cylinder's translation and orientation.

(3) Model-experiment comparison shows that the model well predicts the cylinder's translation and orientation. However, the performance of the numerical model for $\chi=0$ is not as good as for $\chi\neq0$.

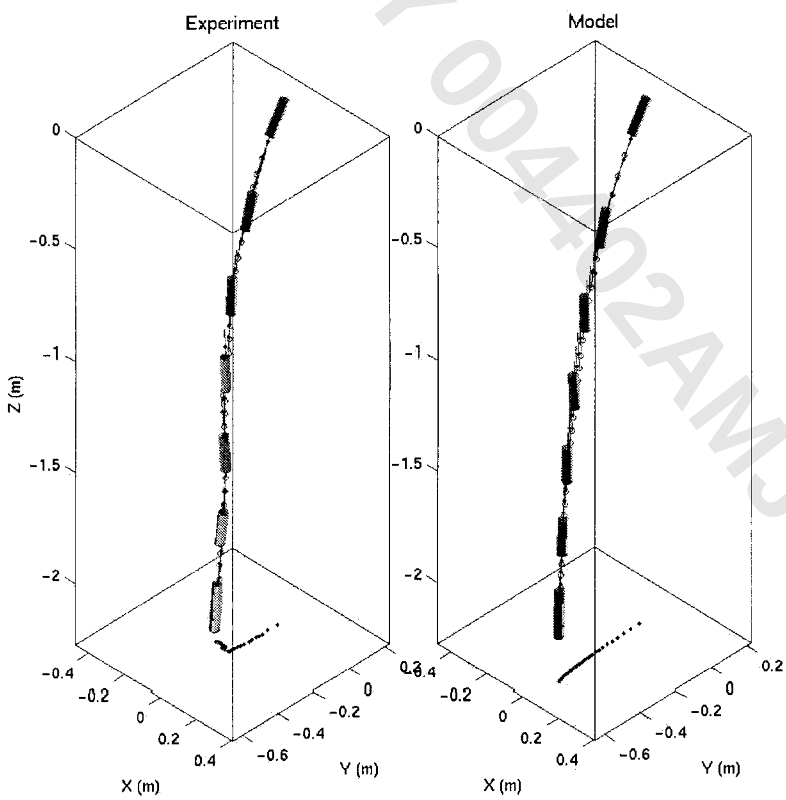

Fig. 4 Movement of Cylinder #1 ($L=15.20$ cm, $\bar{\rho}=1.69$ g cm^{-3}) with $\chi=0.74$ m and drop angle 45 deg obtained from (*a*) experiment, and (*b*) recursive model

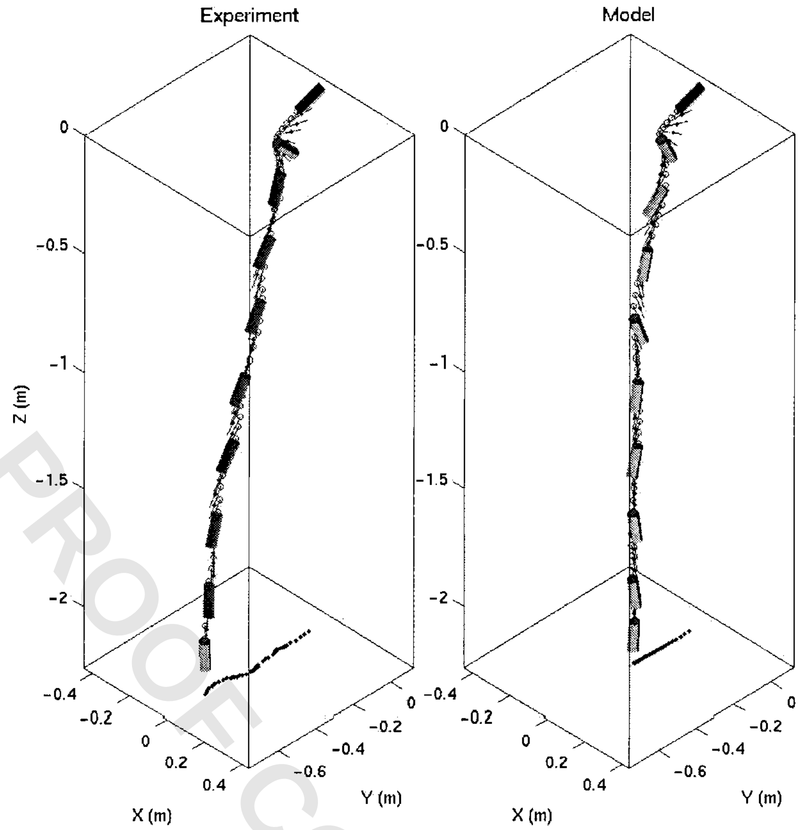

Fig. 5 Movement of Cylinder #2 ($L = 12.10$ cm, $\bar{\rho} = 1.67$ g cm^{-3}) with $\chi = -1.00$ cm and drop angle 30 deg obtained from (*a*) experiment, and (*b*) recursive model

Acknowledgments

This research was funded by the Office of Naval Research Coastal Geosciences Program (grant N0001403WR20178), the Naval Oceanographic Office, and the Naval Postgraduate School supported this study.

References

[1] White, F. M., 1974, *Viscous Fluid Flow*, 1st Ed., McGraw-Hill, New York.
[2] Chu, P. C., Gilless, A. F., Fan, C., and Fleischer, P., 2002, "Hydrodynamical Characteristics of a Falling Cylinder in the Water Column," *Advances in Fluid Mechanics*, **4**, M. Rahman, R. Verhoeven, and C. A. Brebbia, eds., WIT Press, Southampton, UK, pp. 163–181.

PROOF COPY 004402AMJ